The forward-backward asymmetry in electron-positron annihilation

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Introduction: **Electroweak precision physics**

I.: **Higher order corrections**

II: **Infrared-safe definition of the observable**

III: **Outline of the calculation**

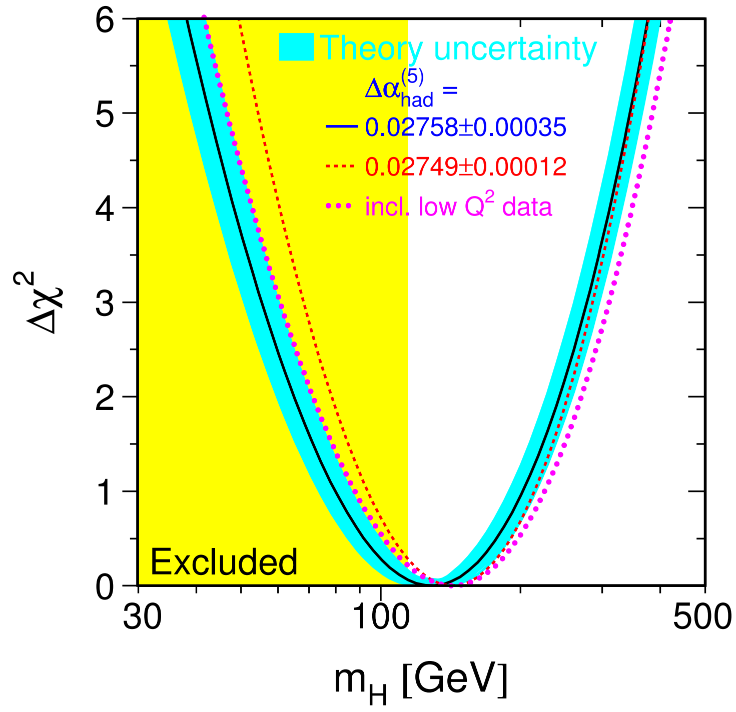
IV.: **Results**

The Standard Model and the Higgs boson

Our current paradigma: The Standard Model

The Higgs boson: The Standard Model predicts a scalar particle, which gives rise to the mass of all other particles.
- yet to be discovered -

Up to the Higgs boson manifests itself only through quantum corrections!



(Electroweak Working Group, hep-ex/0509008.)

Electroweak precision physics

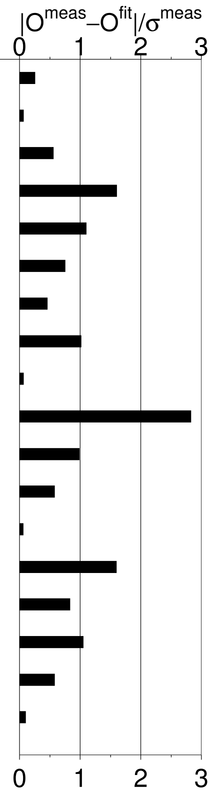
Precision observables allow us to extract the values of the five input parameters for the Standard model at the Z-pole.

Input parameters are:
$\alpha(m_Z^2)$, $\alpha_s(m_Z^2)$, m_Z, m_t, m_H.

Check how individual measurements agree with the results of this fit.

The forward-backward asymmetry for b-quarks shows the largest pull.

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ_{had}^0 [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21562
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.389
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.5



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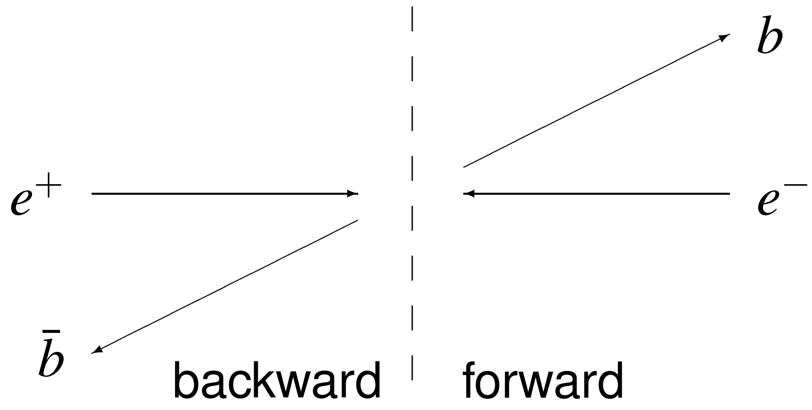
The forward-backward asymmetry



A first definition of the forward-backward asymmetry:

$$A_{FB} \quad = \quad \frac{N_F - N_B}{N_F + N_B}$$

But: Free quarks are not observed, instead hadronic jets are seen in the detector !

Perturbation theory

Due to the smallness of the coupling constants α and α_s, we may compute observables at high energies reliable in perturbation theory,

$$\langle O \rangle = \langle O \rangle_{LO} + \frac{\alpha_s}{2\pi} \langle O \rangle_{NLO} + \left(\frac{\alpha_s}{2\pi}\right)^2 \langle O \rangle_{NNLO} + \ldots$$

provided that the observable is infrared-safe!

In particular, it is required that they do not change value, if infinitessimal soft or collinear particles are added.

$$O_{n+l}(p_1, \ldots, p_{n+l}) \rightarrow O_n(p'_1, \ldots, p'_n),$$

The forward-backward asymmetry is measured experimentally with a precision at the per cent level.

To match this precision the inclusion of QCD corrections in a theoretical calculation is mandatory.

Prior art

Calculation of the NNLO QCD corrections to the forward-backward asymmetry in massless QCD:

$$A_{FB} = A_{FB}^{(0)}\left(1+\frac{\alpha_s}{2\pi}B_{FB}+\left(\frac{\alpha_s}{2\pi}\right)^2 C_{FB}\right)+O\left(\alpha_s^3\right),$$

- G. Altarelli and B. Lampe, 1993;
- V. Ravindran and W. L. van Neerven, 1998;
- S. Catani and M. H. Seymour, 1999.

NLO corrections including mass corrections:

J. Jersak, E. Laermann, and P. M. Zerwas, 1981; J. G. Körner, G. Schuler, G. Kramer, and B. Lampe, 1986; A. B. Arbuzov, D. Y. Bardin, and A. Leike, 1992; A. Djouadi, B. Lampe, and P. M. Zerwas, 1995; B. Lampe, 1996;

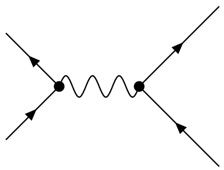
Partial results for mass corrections at NNLO:

W. Bernreuther, A. Brandenburg, and P. Uwer, 2000; W. Bernreuther *et al.*, 2006;

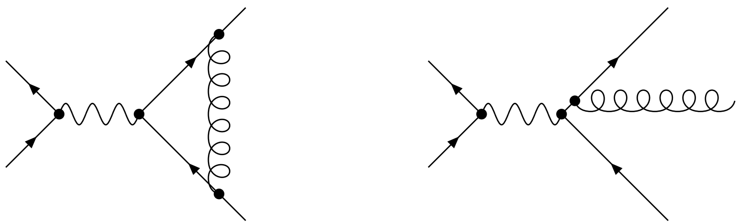
Diagrams

Some examples of diagrams contributing to the various orders in perturbation theory:

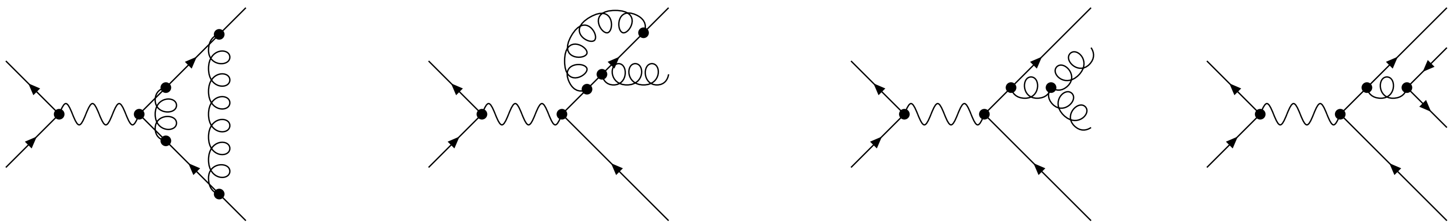
LO:



NLO:



NNLO:



Purely virtual diagrams cancel in the correction to the asymmetry!

Definitions used in the literature

How to define the direction of the b-quark in the presence of additional partons?

- Define the direction by the momentum of the quark.
- Use the thrust axis as direction.

How to treat the $bb\bar{b}\bar{b}$ final state if two b-quarks are tagged?

- Count it once.
- Count it twice.

The experimental analysis seems to have used the thrust axis and counted $bb\bar{b}\bar{b}$ final states with weigth two.

Infrared finiteness

Catani ans Seymour have shown, that none of the combinations thrust axis/ quark axis and weight two/ weight one yields an infrared finite observable.

The divergence is proportional to

$$\int_0^1 dz\, P_{q \to q q \bar{q}}(z) \,\ln \frac{Q^2}{m_b^2}$$

To absorb this divergence one can introduce a b-quark fragmentation function.
This brings along additional uncertainties related to non-perturbative physics.

Questions

Can the introduction of the fragmentation function and dependence on non-perturbative physics be avoided ?

How to define the forward-backward asymmetry in an infrared-safe way ?

What about a jet axis ?

Jet algorithms

The most fine-grained look at hadronic events consistent with infrared safety is given by classifying the particles into jets.

Ingredients:

- a resolution variable y_{ij} where a smaller y_{ij} means that particles i and j are "closer";
- a combination procedure which combines two four-momenta into one;
- a cut-off y_{min} which provides a stopping point for the algorithm.

A typical algorithm:

- for each pair i, j, calculate y_{ij}
- select pair with smallest y_{ij}; if $y_{ij} < y_{min}$, combine i and j
- repeat until the smallest $y_{ij} > y_{min}$

The Durham algorithm

Example: The Durham or $k_\perp$-algorithm for partons, whose flavour is not detected.

(Dokshitzer, 1991)

Resolution variable:

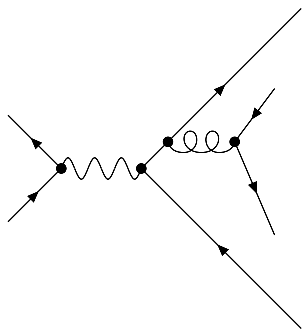
$$y_{ij}^{DURHAM} = \frac{2(1-\cos\theta_{ij})}{Q^2} \min(E_i^2, E_j^2)$$

Combination procedure:

$$p_{(ij)}^{\mu} = p_i^{\mu} + p_j^{\mu}.$$

Jets with flavour

The Durham algorithm is not infrared-safe for jets with flavour, since at order α_s^2 a soft gluon can split into a soft $q\bar{q}$ pair.



The Durham measure

$$y_{ij}^{DURHAM} = \frac{2(1-\cos\theta_{ij})}{Q^2}\,\mathrm{min}(E_i^2, E_j^2)$$

assumes that parton emission has a soft and a collinear divergence.

However, there is no soft divergence in the $g \to q\bar{q}$ splitting.

The flavour-$k_\perp$ algorithm

In order to account for tagged flavours modify the Durham measure

$$y_{ij}^{DURHAM} = \frac{2(1-\cos\theta_{ij})}{Q^2}\,\mathrm{min}(E_i^2, E_j^2)$$

towards

$$y_{ij}^{flavour} = \frac{2(1-\cos\theta_{ij})}{Q^2} \times \begin{cases} \mathrm{min}(E_i^2, E_j^2), & \text{softer of } i,\ j \text{ is flavourless,} \\ \mathrm{max}(E_i^2, E_j^2), & \text{softer of } i,\ j \text{ is flavoured.} \end{cases}$$

This yields an infrared-safe definition of jets if flavours are tagged.

Banfi, Salam and Zanderighi, (2006).

Definition of the forward-backward asymmetry

- Assign flavour number $+1$ to a b-quark and -1 to a $\bar{b}$-quark.
  All other particles have flavour number zero.

- Cluster particles into jets, using the flavour-$k_\perp$ algorithm.

- If two particles are combined, the flavour numbers are added.

- Select two jet events, where one jet has flavour number > 0.

- The jet axis of this jet defines the direction relevant to the forward-backward asymmetry.

Calculation of the NLO and NNLO corrections

To compute for this definition the NLO and NNLO corrections, a general purpose program for NNLO corrections to $e^+e^- \longrightarrow$ 2 jets is used.

S.W., 2006.

The relevant matrix elements are known for a long time.

T. Matsuura and W. L. van Neerven, 1988; T. Matsuura, S. C. van der Marck, and W. L. van Neerven, 1989; G. Kramer and B. Lampe, 1987; R. K. Ellis, D. A. Ross, and A. E. Terrano, 1981; A. Ali *et al.*, 1979;

Difficulty: Cancellation of IR divergences.

General methods at NLO

Fully differential NLO Monte Carlo programs need a general method to handle the cancelation of infrared divergencies.

- Phase space slicing
  - e^+e^-: W. Giele and N. Glover, (1992)
  - initial hadrons: W. Giele, N. Glover and D.A. Kosower, (1993)
  - massive partons, fragmentation: S. Keller and E. Laenen, (1999)
- Subtraction method
  - residue approach: S. Frixione, Z. Kunzst and A. Signer, (1995)
  - dipole formalism: S. Catani and M. Seymour, (1996)
  - massive partons: L. Phaf and S.W. (2001), S. Catani, S. Dittmaier, M. Seymour and Z. Trócsányi, (2002)

The dipole formalism

The dipole formalism is based on the subtraction method. The NLO cross section is rewritten as

$$\sigma^{NLO} = \int_{n+1} d\sigma^R + \int_n d\sigma^V$$

$$= \int_{n+1} \left(d\sigma^R - d\sigma^A\right) + \int_n \left(d\sigma^V + \int_1 d\sigma^A\right)$$

The approximation $d\sigma^A$ has to fulfill the following requirements:

- $d\sigma^A$ must be a proper approximation of $d\sigma^R$ such as to have the same pointwise singular behaviour in D dimensions as $d\sigma^R$ itself. Thus, $d\sigma^A$ acts as a local counterterm for $d\sigma^R$ and one can safely perform the limit $\varepsilon \to 0$.

- Analytic integrability in D dimensions over the one-parton subspace leading to soft and collinear divergences.

The subtraction method at NNLO

- Singular behaviour
  - Factorization of tree amplitudes in double unresolved limits, Berends, Giele, Cambell, Glover, Catani, Grazzini, Del Duca, Frizzo, Maltoni, Kosower '99
  - Factorization of one-loop amplitudes in single unresolved limits, Bern, Del Duca, Kilgore, Schmidt, Kosower, Uwer, Catani, Grazzini, '99

- Extension of the subtraction method to NNLO Kosower; S.W.; Anastasiou, Melnikov, Petriello; Kilgore; Gehrmann-De Ridder, Gehrmann, Glover, Heinrich; Frixione, Grazzini; Somogyi, Trócsányi and Del Duca;

- Applications:
  - $pp \to W$, Anastasiou, Dixon, Melnikov, Petriello '03,
  - $e^+e^- \to$ 2 jets, Anastasiou, Melnikov, Petriello '04,

The subtraction method at NNLO

Contributions at NNLO:

$$
\begin{aligned}
d\sigma_{n+2}^{(0)} &= \left(\mathcal{A}_{n+2}^{(0)}{}^{*} \mathcal{A}_{n+2}^{(0)} \right) d\phi_{n+2}, \\
d\sigma_{n+1}^{(1)} &= \left(\mathcal{A}_{n+1}^{(0)}{}^{*} \mathcal{A}_{n+1}^{(1)} + \mathcal{A}_{n+1}^{(1)}{}^{*} \mathcal{A}_{n+1}^{(0)} \right) d\phi_{n+1}, \\
d\sigma_{n}^{(2)} &= \left(\mathcal{A}_{n}^{(0)*} \mathcal{A}_{n}^{(2)} + \mathcal{A}_{n}^{(2)*} \mathcal{A}_{n}^{(0)} + \mathcal{A}_{n}^{(1)*} \mathcal{A}_{n}^{(1)} \right) d\phi_{n},
\end{aligned}
$$

Adding and subtracting:

$$
\begin{aligned}
\langle O \rangle_n^{NNLO} &= \int \left(O_{n+2}\, d\sigma_{n+2}^{(0)} - O_{n+1} \circ d\alpha_{n+1}^{(0,1)} - O_n \circ d\alpha_n^{(0,2)} \right) \\
&+ \int \left(O_{n+1}\, d\sigma_{n+1}^{(1)} + O_{n+1} \circ d\alpha_{n+1}^{(0,1)} - O_n \circ d\alpha_n^{(1,1)} \right) \\
&+ \int \left(O_n\, d\sigma_n^{(2)} + O_n \circ d\alpha_n^{(0,2)} + O_n \circ d\alpha_n^{(1,1)} \right).
\end{aligned}
$$

NNLO subtraction terms

The $(n+2)$-parton contribution:

$$\int \left(O_{n+2}\, d\sigma^{(0)}_{n+2} - O_{n+1} \circ d\alpha^{(0,1)}_{n+1} - O_n \circ d\alpha^{(0,2)}_n \right), \qquad d\alpha^{(0,2)}_n = d\alpha^{(0,2)}_{(0,0)n} - d\alpha^{(0,2)}_{(0,1)n}.$$

has to be integrable for all double and single unresolved limits.

The $(n+1)$-parton contribution:

$$\int \left(O_{n+1}\, d\sigma^{(1)}_{n+1} + O_{n+1} \circ d\alpha^{(0,1)}_{n+1} - O_n \circ d\alpha^{(1,1)}_n \right), \qquad d\alpha^{(1,1)}_n = d\alpha^{(1,1)}_{(1,0)\,n} + d\alpha^{(1,1)}_{(0,1)\,n}$$

has to be integrable over single unresolved limits.
In addition, explicit poles in ε have to cancel.

Example: $qgg\bar{q}$ final state for $e^+e^- \longrightarrow$ 2 jets

NNLO subtraction terms for the $(n+2)$-parton configuration:

$$d\alpha^{(0,2)}_{(0,0)} = \frac{1}{2}\left\{\frac{N}{2}C_F\left[A_4^0(1,2,3,4)+A_4^0(1,3,2,4)\right]\right.$$
$$\left. -\frac{1}{2N}C_F\left[A_{4,sc}^0(1,2,3,4)+A_{4,sc}^0(1,3,2,4)\right]\right\}\left|\mathcal{A}_2^{(0)}\right|^2$$

$$d\alpha^{(0,2)}_{(0,1)} = \frac{1}{2}\left\{\frac{N}{2}\left[D_3^0(1,2,3)+D_3^0(1,3,2)+D_3^0(4,2,3)+D_3^0(4,3,2)\right]\right.$$
$$\left. -\frac{1}{2N}\left[A_3^0(1,2,4)+A_3^0(1,3,4)\right]\right\}C_F A_3^0(1',2',3')\left|\mathcal{A}_2^{(0)}\right|^2.$$

Spin-averaged antenna functions

Spin-averaged $qgg\bar{q}$ antenna function obtained from the matrix element $\gamma^* \rightarrow qgg\bar{q}$:

$$\mathcal{A}_4^{(0)}(q_1,g_2,g_3,\bar{q}_4) = eg^2\left[\left(T^2T^3\right)_{14}A_4^{(0)}(q_1,g_2,g_3,\bar{q}_4)+\left(T^3T^2\right)_{14}A_4^{(0)}(q_1,g_3,g_2,\bar{q}_4)\right]$$

$$\left|\mathcal{A}_4^{(0)}\right|^2 = e^2g^4\frac{N(N^2-1)}{4}\left(A_4^{(0)}(2,3),A_4^{(0)}(3,2)\right)\begin{pmatrix} 1-\frac{1}{N^2} & -\frac{1}{N^2} \\ -\frac{1}{N^2} & 1-\frac{1}{N^2}\end{pmatrix}\begin{pmatrix} A_4^{(0)}(2,3) \\ A_4^{(0)}(3,2)\end{pmatrix}$$

Leading-colour antenna function:

$$A_4^0(1,2,3,4) = \left|A_4^{(0)}(2,3)\right|^2/\left|A_2^{(0)}\right|^2$$

Subleading-colour:

$$A_{4,sc}^0(1,2,3,4)+A_{4,sc}^0(1,3,2,4) = \left|A_4^{(0)}(2,3)+A_4^{(0)}(3,2)\right|^2/\left|A_2^{(0)}\right|^2$$

Example: Leading colour $qgg\bar{q}$ antenna function

$$A_4^0(1,2,3,4) =$$

$$\frac{1}{4s_{1234}}\left(\frac{48\,s_{1234}}{s_{234}{}^2}+\frac{32\,s_{1234}}{s_{23}{}^2}+\frac{48\,s_{1234}}{s_{123}{}^2}+\frac{48\,s_{23}-48\,s_{123}+64\,s_{1234}}{s_{12}\,s_{234}}+\frac{-32\,s_{123}\,s_{1234}+16\,s_{123}{}^2-32\,s_{34}\,s_{1234}+16\,s_{34}{}^2+32\,s_{1234}{}^2}{s_{12}\,s_{23}\,s_{234}}\right.$$

$$+\frac{-48\,s_{12}-96\,s_{23}-48\,s_{34}-96\,s_{1234}}{s_{123}\,s_{234}}-\frac{16\,s_{1234}}{s_{34}\,s_{234}}+\frac{-32\,s_{123}\,s_{1234}+16\,s_{123}{}^2-32\,s_{1234}\,s_{234}+16\,s_{234}{}^2+32\,s_{1234}{}^2}{s_{12}\,s_{23}\,s_{34}}+\frac{96}{s_{123}}+\frac{32\,s_{1234}}{s_{12}\,s_{34}}$$

$$-\frac{16\,s_{1234}}{s_{12}\,s_{123}}+\frac{64\,s_{12}\,s_{34}\,s_{1234}}{s_{23}{}^2s_{123}\,s_{234}}+\frac{64\,s_{12}\,s_{1234}-32\,s_{12}{}^2+64\,s_{34}\,s_{1234}-32\,s_{34}{}^2-128\,s_{1234}{}^2}{s_{23}\,s_{123}\,s_{234}}+\frac{16\,s_{23}\,s_{1234}}{s_{34}\,s_{234}{}^2}+\frac{48\,s_{23}-48\,s_{234}+64\,s_{1234}}{s_{123}\,s_{34}}$$

$$+\frac{48\,s_{12}-48\,s_{123}+32\,s_{1234}}{s_{23}\,s_{234}}+\frac{64\,s_{34}\,s_{1234}}{s_{23}\,s_{234}{}^2}+\frac{64\,s_{12}\,s_{1234}}{s_{23}\,s_{123}{}^2}-\frac{64\,s_{34}\,s_{1234}}{s_{23}{}^2s_{234}}+\frac{48\,s_{34}-48\,s_{234}+32\,s_{1234}}{s_{23}\,s_{123}}+\frac{32\,s_{34}{}^2s_{1234}}{s_{23}{}^2s_{234}{}^2}-\frac{64\,s_{12}\,s_{1234}}{s_{23}{}^2s_{123}}$$

$$+\frac{32\,s_{12}{}^2s_{1234}}{s_{23}{}^2s_{123}{}^2}+\frac{16\,s_{23}\,s_{1234}}{s_{12}\,s_{123}{}^2}+\frac{-32\,s_{12}\,s_{1234}+16\,s_{12}{}^2-32\,s_{1234}\,s_{234}+16\,s_{234}{}^2+32\,s_{1234}{}^2}{s_{23}\,s_{123}\,s_{34}}$$

$$+\frac{-32\,s_{23}\,s_{1234}-16\,s_{23}{}^2+48\,s_{34}\,s_{1234}-16\,s_{34}{}^2-64\,s_{1234}{}^2}{s_{12}\,s_{123}\,s_{234}}+\frac{48\,s_{12}\,s_{1234}-16\,s_{12}{}^2-32\,s_{23}\,s_{1234}-16\,s_{23}{}^2-64\,s_{1234}{}^2}{s_{123}\,s_{34}\,s_{234}}$$

$$\left.+\frac{32\,s_{23}\,s_{1234}{}^2+16\,s_{23}{}^2s_{1234}+32\,s_{1234}{}^3}{s_{12}\,s_{123}\,s_{34}\,s_{234}}+\frac{-32\,s_{23}\,s_{1234}+16\,s_{123}\,s_{1234}-32\,s_{1234}{}^2}{s_{12}\,s_{34}\,s_{234}}+\frac{-32\,s_{23}\,s_{1234}+16\,s_{1234}\,s_{234}-32\,s_{1234}{}^2}{s_{12}\,s_{123}\,s_{34}}+\frac{96}{s_{234}}\right)$$

Spin correlations

In the collinear limit spin correlations remain:

$$A_\mu \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} A_\nu, \qquad \text{where } k_\perp = (1-z)p_i + z p_j - (1-2z)\frac{y}{1-y} p_k.$$

Let φ be the azimuthal angle of p_i around $p_i + p_j$. Then

$$A_\mu \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} A_\nu \quad \sim \quad C_0 + C_2 \cos(2\varphi + \alpha).$$

One can perform the average with two points:

$$\varphi, \qquad \varphi + \frac{\pi}{2},$$

while all other coordinates remain fixed.

Phase space generation

Dimension of phase space for n final state particles: $3n-4$.

Split the phase space into different channels, according to which invariants are the smallest.

For each channel, use a parameterization such that φ is along a coordinate axis:

$$\begin{aligned} d\phi_{n+1} &= d\phi_n \; d\phi_{dipole}, \\ & d\phi_{dipole} = \frac{s_{ijk}}{32\pi^3} (1-y)\, dy\, dz\, d\varphi. \end{aligned}$$

Construct the momenta of the $(n+1)$ event from the ones of the n parton event and the values of y, z and φ.

Numerical results for the forward-backward asymmetry of b-quarks

Perturbative expansion:

$$A_{FB} = A_{FB}^{(0)} \left(1 + \frac{\alpha_s}{2\pi} B_{FB} + \left(\frac{\alpha_s}{2\pi}\right)^2 C_{FB} \right) + \mathcal{O}\left(\alpha_s^3\right),$$

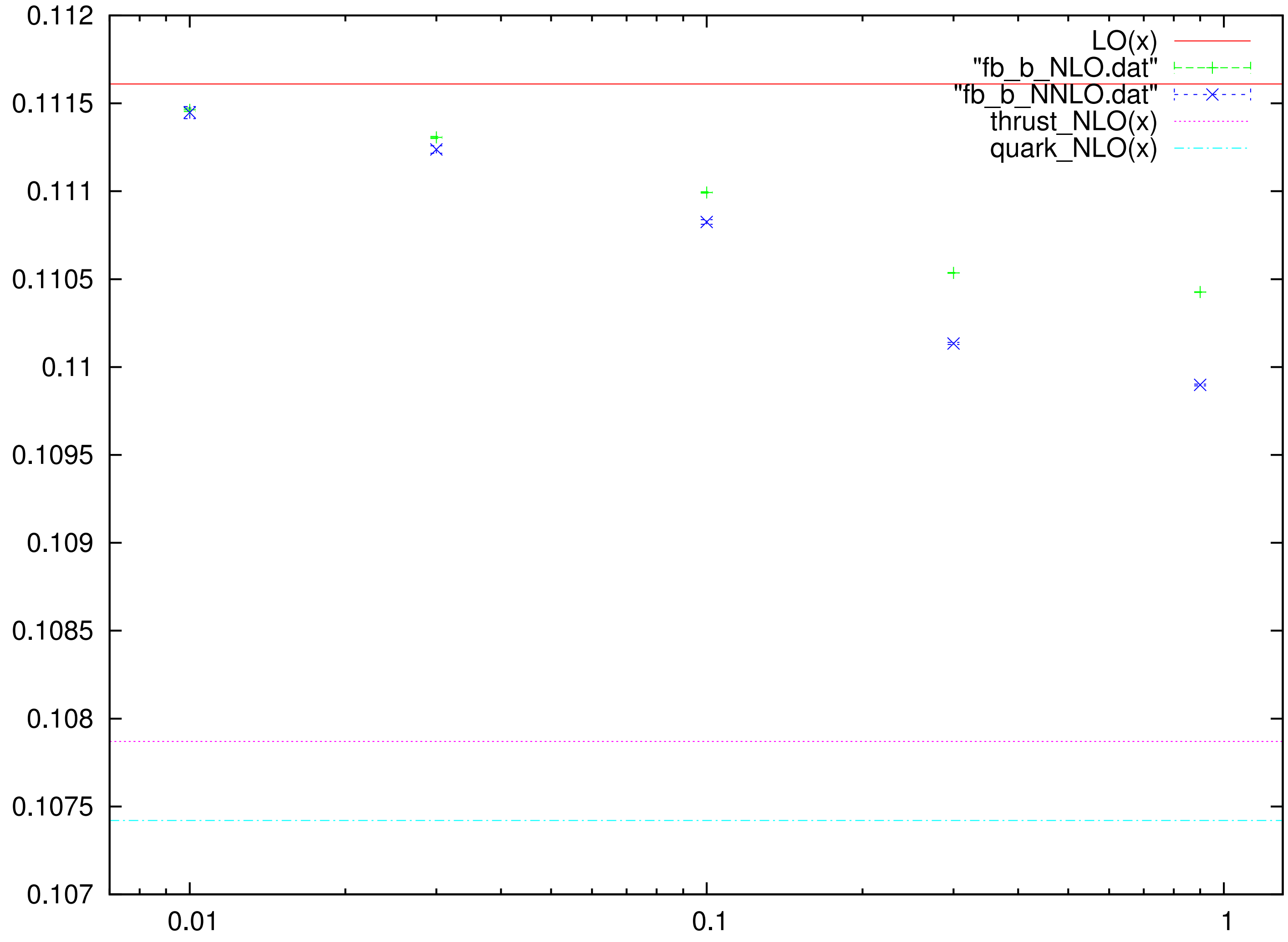
Select two-jet events defined by the flavour-$k_\perp$ algorithm and a given y_{cut}.
Leading order result independent of y_{cut}:

$$A_{FB,b}^{(0)} = 0.11161.$$

QCD corrections:

y_{cut}	$B_{FB,b}$	$C_{FB,b}$
0.01	-0.070 ± 0.005	-0.4 ± 0.8
0.03	-0.145 ± 0.003	-1.7 ± 0.5
0.1	-0.294 ± 0.002	-4.3 ± 0.3
0.3	-0.512 ± 0.001	-10.2 ± 0.1
0.9	-0.565 ± 0.001	-13.4 ± 0.1

Plot



0.112
0.1115
0.111
0.1105
0.11
0.1095
0.109
0.1085
0.108
0.1075
0.107
0.01
0.1
1
LO(x)
"fb_b_NLO.dat"
"fb_b_NNLO.dat"
thrust_NLO(x)
quark_NLO(x)

Numerical results for the forward-backward asymmetry of c-quarks

Perturbative expansion:

$$A_{FB} = A_{FB}^{(0)}\left(1 + \frac{\alpha_s}{2\pi}B_{FB} + \left(\frac{\alpha_s}{2\pi}\right)^2 C_{FB}\right) + \mathcal{O}\left(\alpha_s^3\right),$$

Select two-jet events defined by the flavour-$k_\perp$ algorithm and a given y_{cut}.
Leading order result independent of y_{cut}:

$$A_{FB,c}^{(0)} = 0.08003.$$

QCD corrections:

y_{cut}	$B_{FB,c}$	$C_{FB,c}$
0.01	-0.070 ± 0.005	-0.5 ± 0.7
0.03	-0.145 ± 0.003	-2.1 ± 0.5
0.1	-0.294 ± 0.002	-4.8 ± 0.2
0.3	-0.513 ± 0.001	-12.1 ± 0.2
0.9	-0.565 ± 0.001	-15.9 ± 0.1

Summary

- The forward-backward asymmetry shows the largest discrepancy in a fit of the Standard Model parameter.
- Experimental analysis based on an infrared-unsafe definition.
- Infrared-safe definition of the forward-backward asymmetry.
- Calculation of the NLO and NNLO QCD corrections.
- The corrections are small, useful observable also for a future linear collider.