NLO QCD corrections to top-pair production $+$ jet

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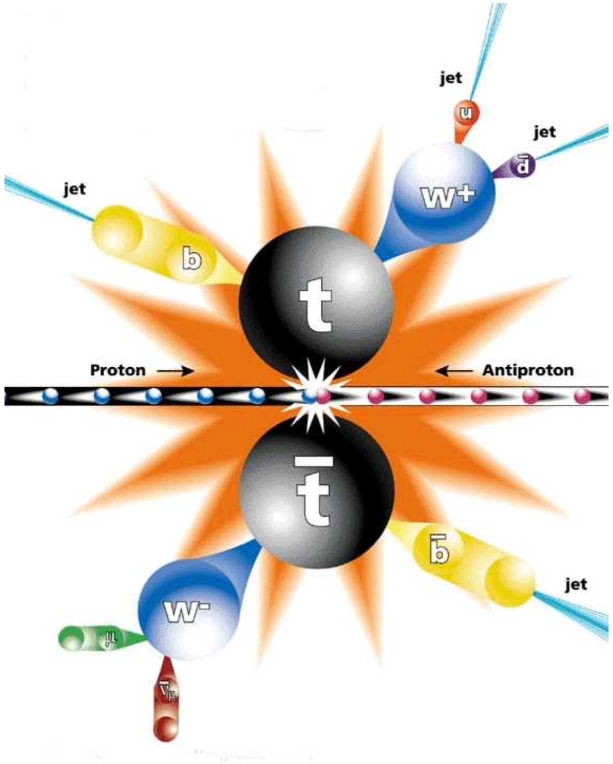
Introduction: **Top physics**

I.: **Virtual corrections**

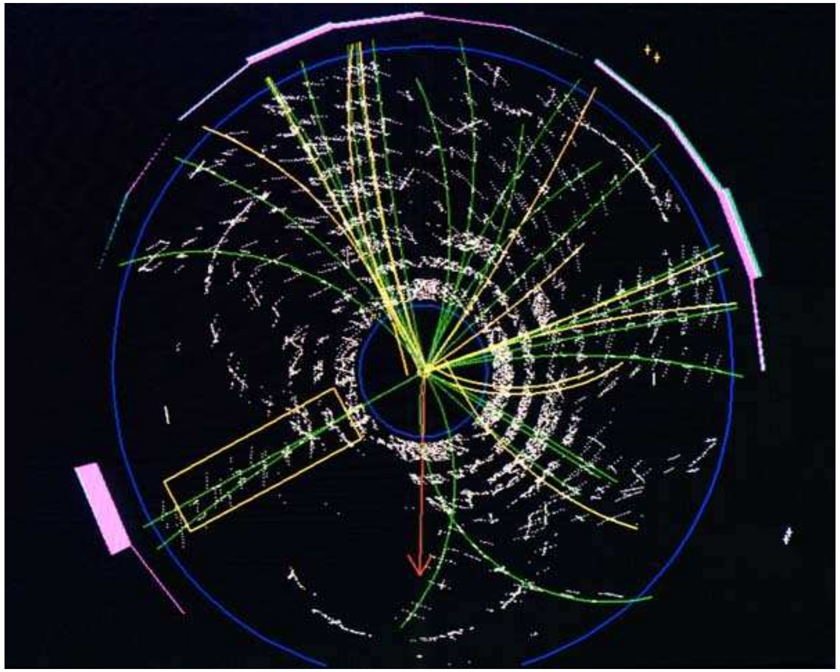
II: **Real corrections**

III: **Results**

LHC physics



A schematic view of a top-pair event.



A $t\bar{t}$ event from CDF.

Jets: A bunch of particles moving in the same direction

Phenomenological relevance of $t\bar{t}$ + jet

- Interesting signal process
  - Top quark physics plays an important role at the LHC.
  - Large fraction of inclusive $t\bar{t}$ events are due to $t\bar{t}$ + jet.
  - Search for anomalous couplings.
  - Measurement of the forward-backward charge asymmetry at the Tevatron.

- Important background process for Higgs searches at the LHC:
  - Vector-boson-fusion: $pp(WW \to H) \to H + 2$ jets.
  - Associated Higgs production: $pp \to t\bar{t}H$.

- Important background process for SUSY searches at the LHC.

The forward-backward charge asymmetry

Forward-backward charge asymmetry in $q\bar{q} \to t\bar{t}(+\text{jets})$

Origin: Interference of C-odd with C-even parts.

$q\bar{q} \to t\bar{t}$: asymmetry appears first at NLO (Kühn, Rodrigo '98).
A_{FB} @ NLO not feasible in the near future (requires σ_{NNLO})

$q\bar{q} \to t\bar{t} + \text{jet}$: asymmetry is LO effect (Halzen, Hoyer, Kim, ' 87).
A_{FB} @ NLO can be deduced from σ_{NLO}.

LO-study for the Tevatron Bowen, Ellis, Rainwater, '05

“Technical” inportance of $t\bar{t} + $ jet

Benchmark process for one-loop calculations at the LHC.

Significant complexity due to:

- All partons coloured.
- Additional mass scale m_t.
- Complicated infra-red structure.
- Many sub-processes, many diagrams, ...

Test ground for the development of new methods for one-loop calculations.

The master formula for the calculation of observables

$$\langle O \rangle = \underbrace{\frac{1}{2K(s)}}_{\text{flux factor}} \underbrace{\frac{1}{(2J_1+1)}\frac{1}{(2J_2+1)}}_{\text{average over initial spins}} \sum_n \underbrace{\int d\phi_{n-2}}_{\text{integral over phase space}} O(p_1,\ldots,p_n) \sum_{\text{helicity}} \underbrace{|\mathcal{A}_n|^2}_{\text{amplitude}}$$

Perturbative expansion of the amplitude (LO, NLO):

$$|\mathcal{A}_n|^2 = \underbrace{\mathcal{A}_n^{(0)\,*}\mathcal{A}_n^{(0)}}_{Born} + \underbrace{\left(\mathcal{A}_n^{(0)\,*}\mathcal{A}_n^{(1)} + \mathcal{A}_n^{(1)\,*}\mathcal{A}_n^{(0)}\right)}_{virtual},$$

$$|\mathcal{A}_{n+1}|^2 = \underbrace{\mathcal{A}_{n+1}^{(0)\,*}\mathcal{A}_{n+1}^{(0)}}_{real}.$$

The virtual corrections

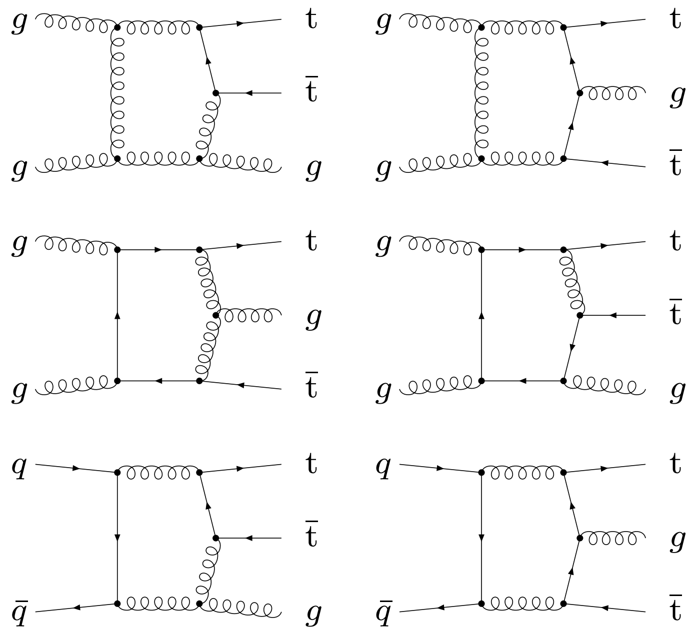
Main problem:
Numerically fast and stable evaluation of tensor integrals:

$$\int d^D k \frac{k^\mu k^\nu k^\rho \dots}{\left(k^2 - m_0^2\right)\left((k+p_1)^2 - m_1^2\right)\dots}$$

Many one-loop diagrams:
~ 350 for $gg \longrightarrow t\bar{t}g$,
~ 100 for $q\bar{q} \longrightarrow t\bar{t}g$.

High degree of automatisation needed!

Most complicated diagrams:
Pentagons



The virtual corrections

Two independent calculations of the virtual corrections:

Calculation 1:

- Diagram generation with FeynArts Küblbeck, Böhm, Denner '90, Hahn '01
- Symbolic part: Mathematica, numerics: Fortran
- Reduction of pentagons Denner, Dittmaier '02
- Analytical extraction of soft/collinear singularities Beenakker et al. '02, Dittmaier '03
- Treatment of critical phase space regions Denner, Dittmaier '05

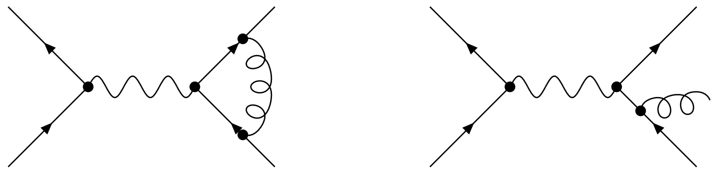
Calculation 2:

- Diagram generation with Qgraf Nogueira '93
- Symbolic part: Form Vermaseren '00, numerics: C++
- Reduction of pentagons Giele, Glover '04
- Treatment of critical phase space regions Giele, Glover, Zanderighi '04

Infrared divergences and the Kinoshita-Lee-Nauenberg theorem

In addition to ultraviolet divergences, loop integrals can have infrared divergences.

For each IR divergence there is a corresponding divergence with the opposite sign in the real emission amplitude, when particles becomes soft or collinear (e.g. unresolved).



The Kinoshita-Lee-Nauenberg theorem: Any observable, summed over all states degenerate according to some resolution criteria, will be finite.

General methods at NLO

Fully differential NLO Monte Carlo programs need a general method to handle the cancelation of infrared divergencies.

- Phase space slicing
  - e^+e^-: W. Giele and N. Glover, (1992)
  - initial hadrons: W. Giele, N. Glover and D.A. Kosower, (1993)
  - massive partons, fragmentation: S. Keller and E. Laenen, (1999)

- Subtraction method
  - residue approach: S. Frixione, Z. Kunzst and A. Signer, (1995)
  - dipole formalism: S. Catani and M. Seymour, (1996)
  - massive partons: L. Phaf and S.W. (2001), S. Catani, S. Dittmaier, M. Seymour and Z. Trócsányi, (2002)

The dipole formalism

The dipole formalism is based on the subtraction method. The NLO cross section is rewritten as

$$\sigma^{NLO} = \int_{n+1} d\sigma^R + \int_n d\sigma^V$$

$$= \int_{n+1} \left(d\sigma^R - d\sigma^A\right) + \int_n \left(d\sigma^V + \int_1 d\sigma^A\right)$$

The approximation $d\sigma^A$ has to fulfill the following requirements:

- $d\sigma^A$ must be a proper approximation of $d\sigma^R$ such as to have the same pointwise singular behaviour in D dimensions as $d\sigma^R$ itself. Thus, $d\sigma^A$ acts as a local counterterm for $d\sigma^R$ and one can safely perform the limit $\varepsilon \to 0$.

- Analytic integrability in D dimensions over the one-parton subspace leading to soft and collinear divergences.

The subtraction terms

The approximation term $d\sigma^A$ is given as a sum over dipoles:

$$d\sigma^A = \sum_{pairs\ i,j} \sum_{k \neq i,j} \mathcal{D}_{ij,k}.$$

Each dipole contribution has the following form:

$$\mathcal{D}_{ij,k} = -\frac{1}{2p_i \cdot p_j} \mathcal{A}_n^{(0)\,*}\left(p_1, \ldots, \tilde{p}_{(ij)}, \ldots, \tilde{p}_k, \ldots\right) \frac{\mathbf{T}_k \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} V_{ij,k} \mathcal{A}_n^{(0)}\left(p_1, \ldots, \tilde{p}_{(ij)}, \ldots, \tilde{p}_k, \ldots\right).$$

- Colour correlations through $\mathbf{T}_k \cdot \mathbf{T}_{ij}$.
- Spin correlations through $V_{ij,k}$.

The real corrections

Two independent calculations of the real corrections minus subtractions:

Calculation 1:

- Helicity amplitudes calculated numerically with recursion relations Berends, Giele '88
- Automated subtraction terms S.W. '05

Calculation 2:

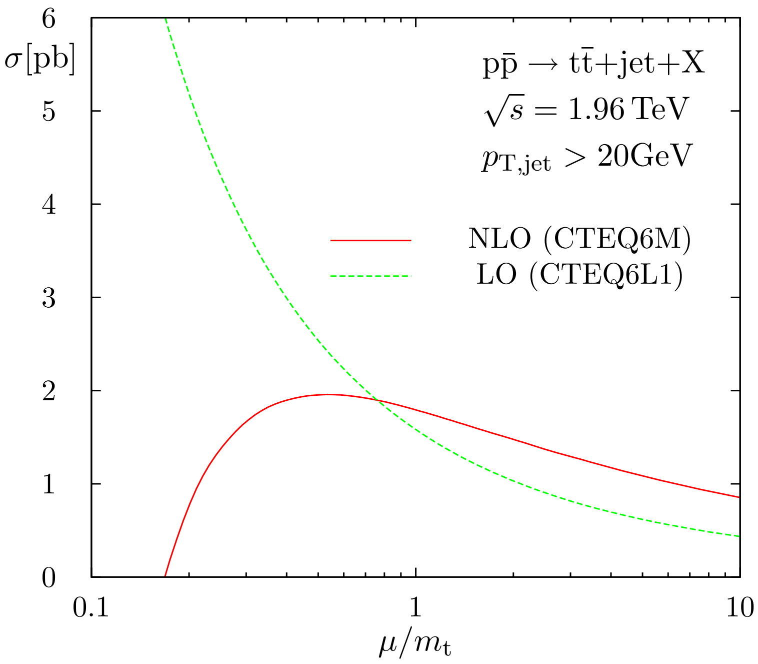
- Analytical expressions for helicity amplitudes and/or Madgraph Stelzer, Long '94
- Semi-automated subtraction terms

Numerical results on $t\bar{t}+$ jet production

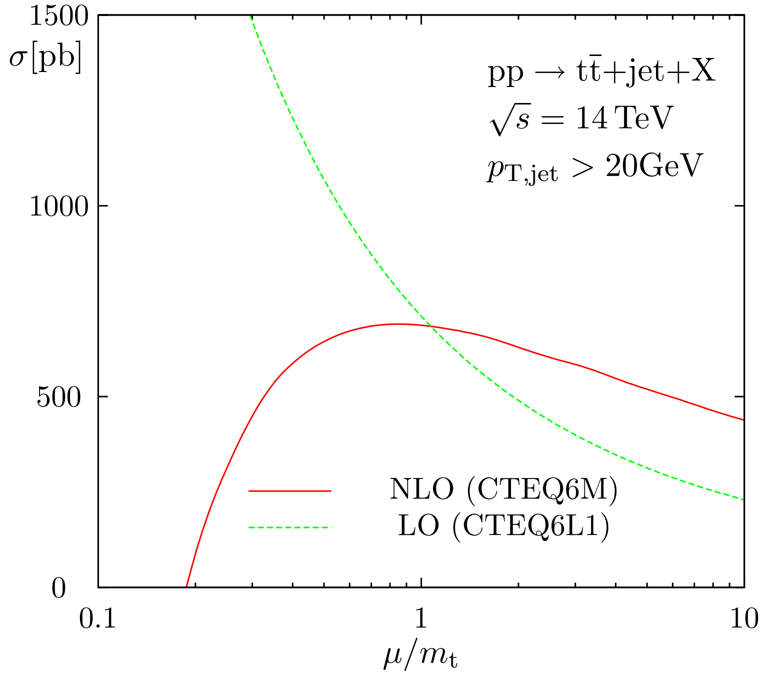
Dependence of the cross section on renormalisation and factorisation scale:

Leading order is proportional to α_s^3 !

Tevatron:

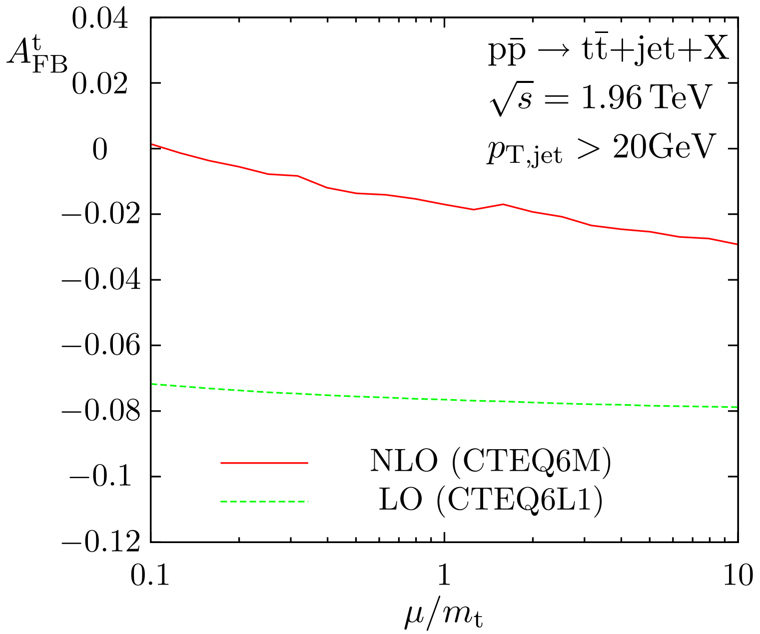


LHC:



Jet definition: $k_\perp$-algorithm with $R = 1$ applied to particles other than t or $\bar{t}$.

Results on the forward-backward asymmetry



$$\sigma^{\pm} = \sigma(y_t > 0) \pm \sigma(y_t < 0),$$

$$A^t_{FB,LO} = \frac{\sigma^-_{LO}}{\sigma^+_{LO}},$$

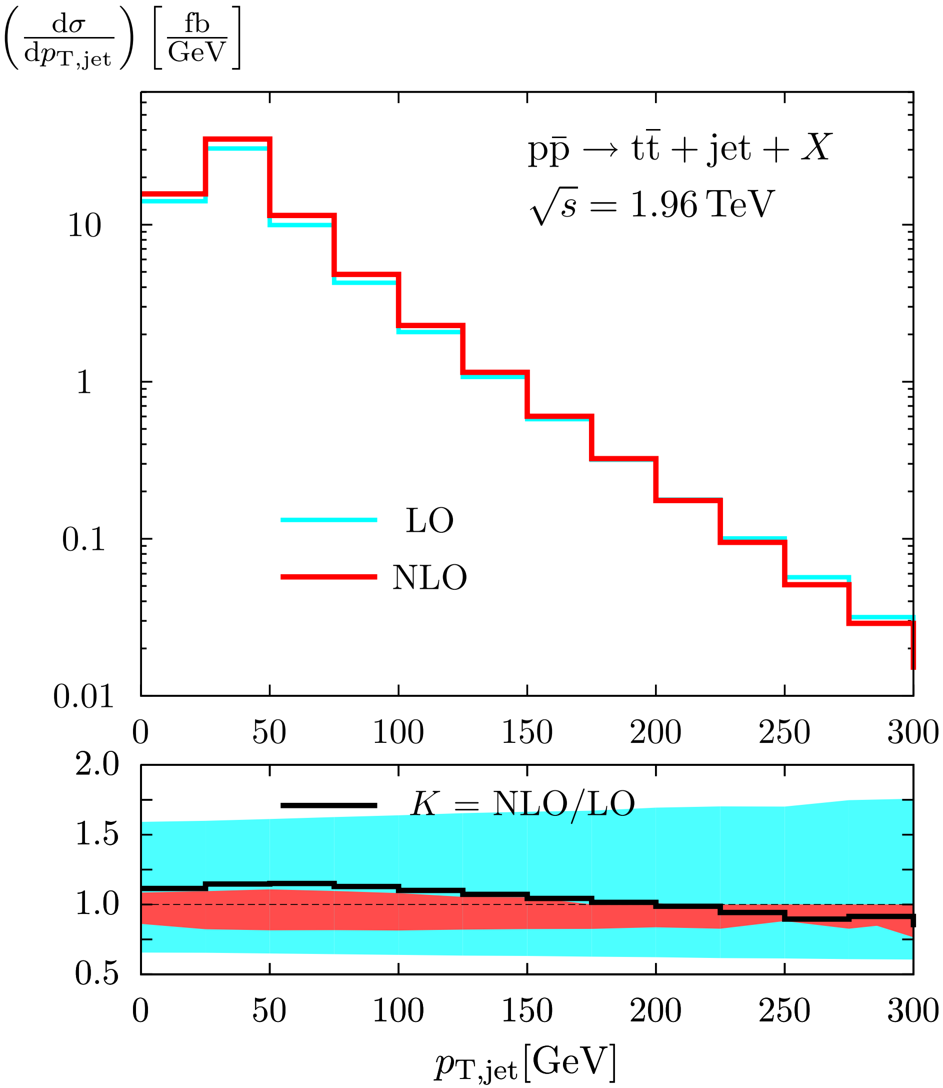
$$A^t_{FB,NLO} = \frac{\sigma^-_{LO}}{\sigma^+_{LO}} \left(1 + \frac{\delta\sigma^-_{NLO}}{\sigma^-_{LO}} - \frac{\delta\sigma^+_{NLO}}{\sigma^+_{LO}}\right).$$

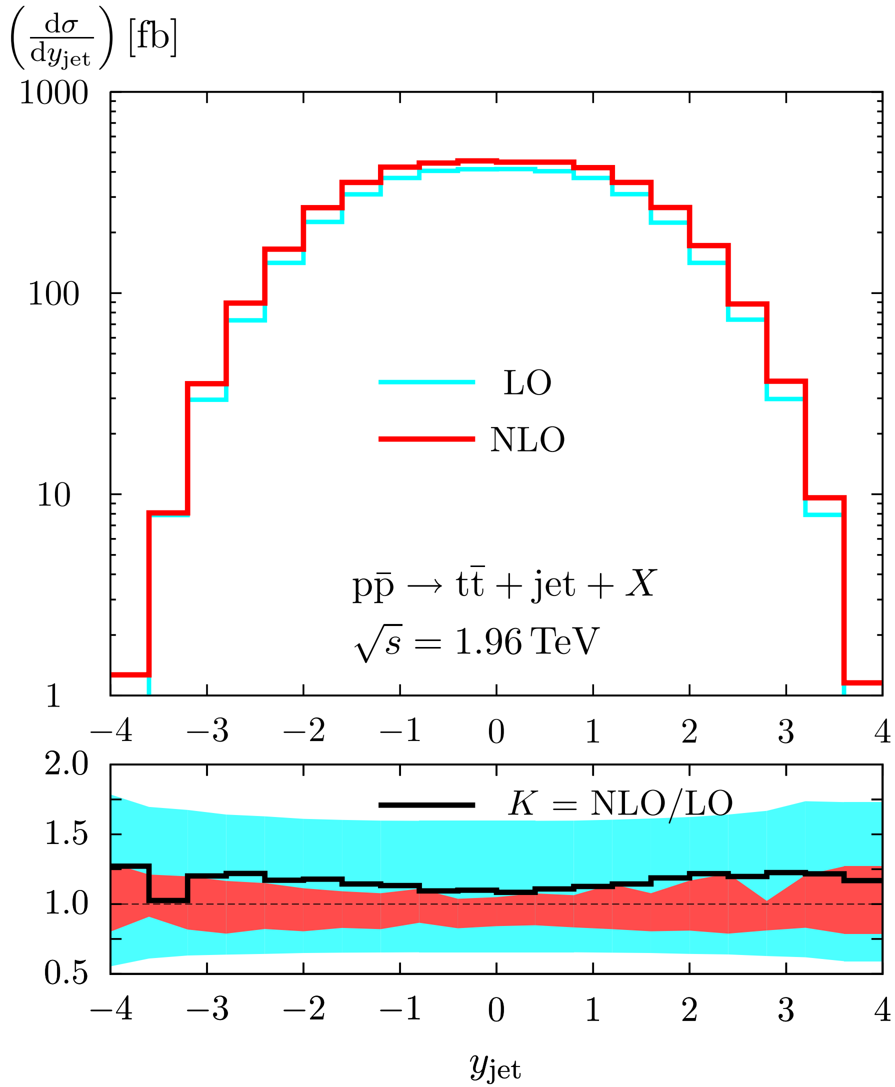
$(\mu = \mu_{ren} = \mu_{fact})$

- $A^t_{FB,LO} = O(\alpha_s^0)$, i.e. no dependence on μ_{ren}
  mild dependence on $\mu_{fact} \ll$ theoretical uncertainty !

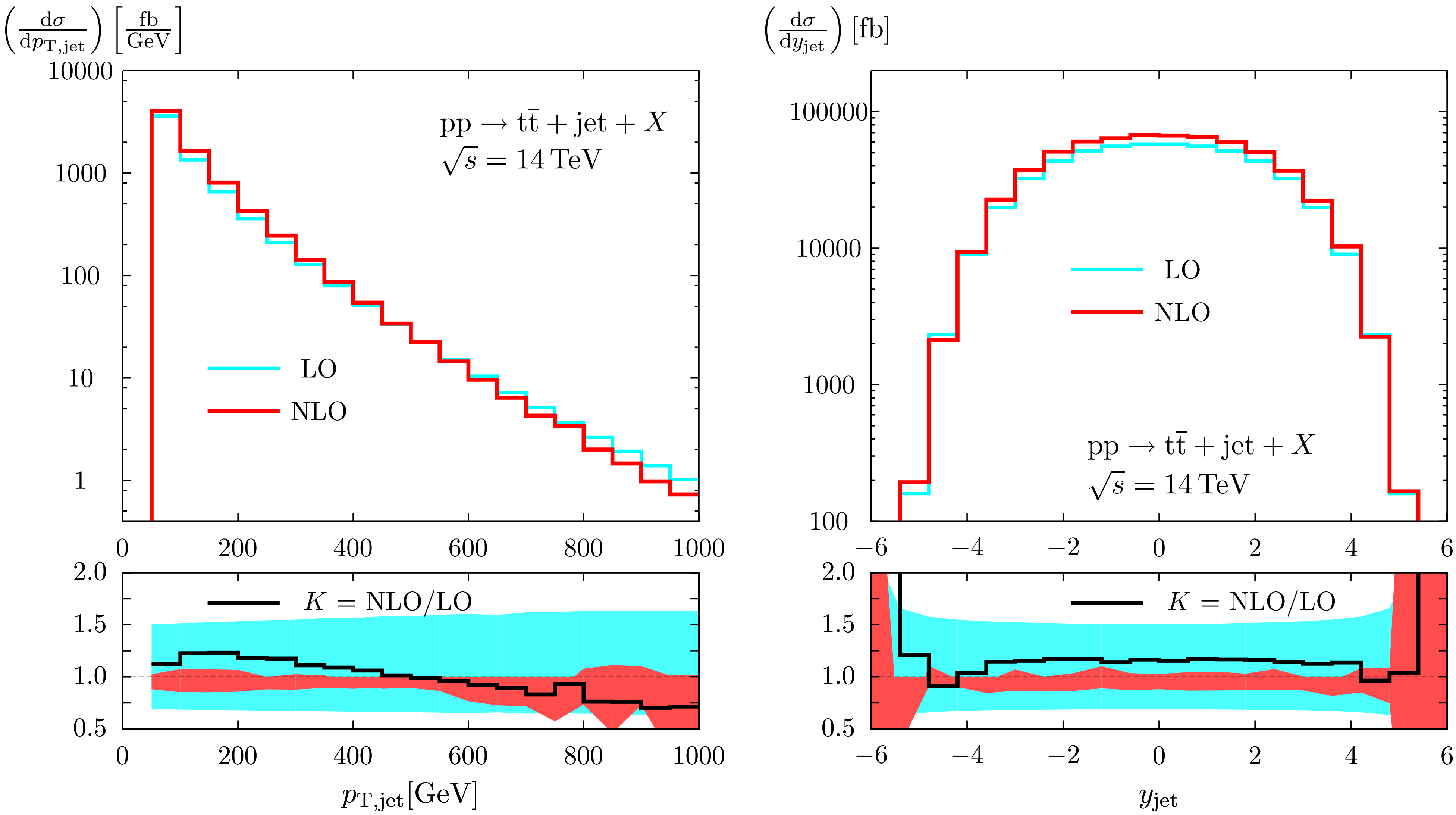
- $A^t_{FB,NLO}$ depends on μ_{fact} and μ_{ren}
  asymmetry almost washed out by scale dependence.

Preliminary results: Distribution of the additional jet at the Tevatron

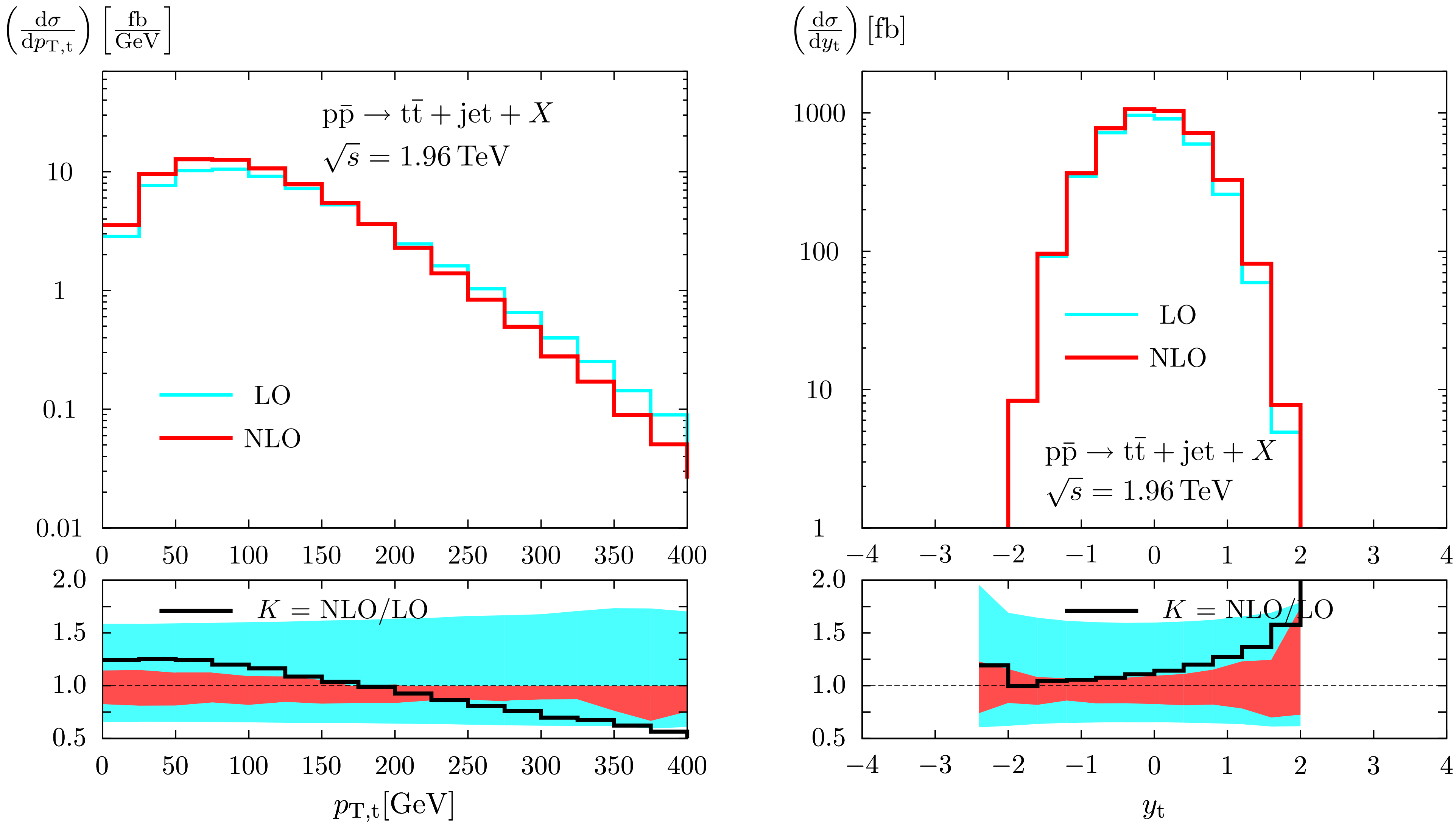




Preliminary results: Distribution of the additional jet at the LHC



Preliminary results: Distribution of the top quark at the Tevatron



Summary

Top-pair production $+$ jet:

- Study of top-quark properties at LHC and Tevatron.

- Important background process for Higgs and Susy searches.

NLO QCD corrections:

- NLO corrections stabilise LO cross section

- Forward-backward asymmetry receives large NLO corrections

- Differential distributions