NLO partonic event generators

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I. **Motivation**:
Merging the benefits of event generators and exact calculations

II. **A general algorithm**:
Generating unweighted events at NLO

III. **Outlook**

JHEP 08, (2001) 028; hep-ph/0106146

Precise theoretical predictions

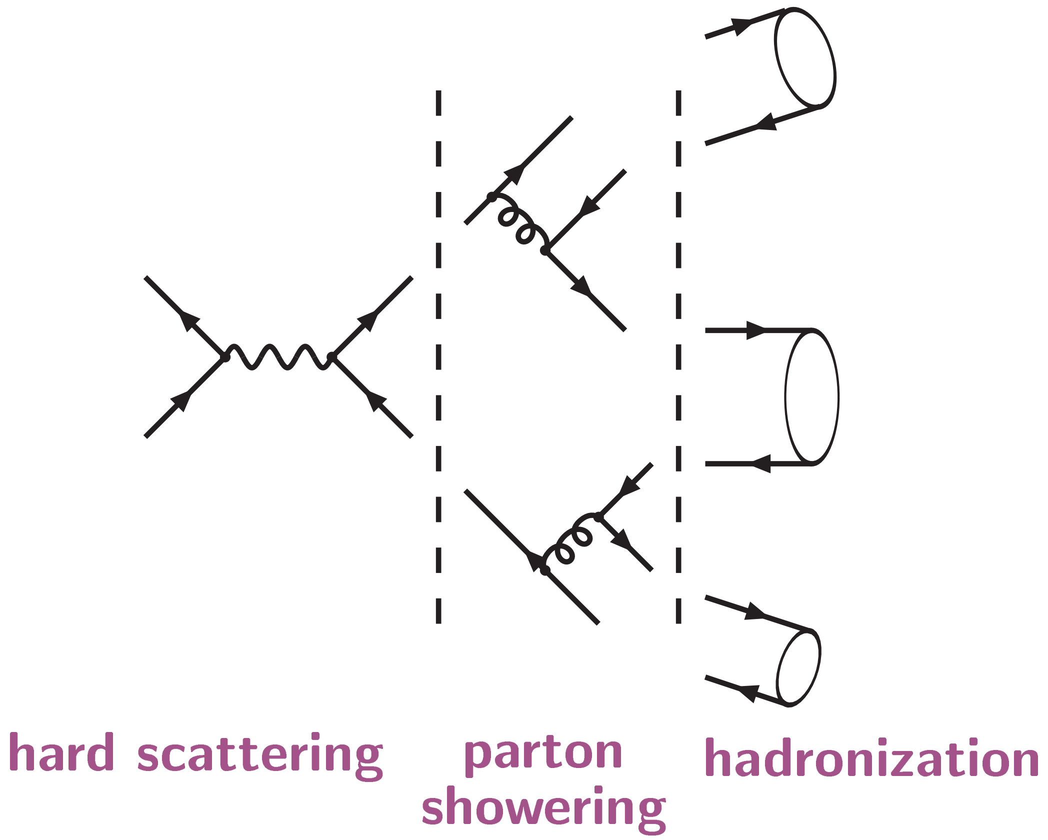
As high energy experiments move towards a new generation of colliders, accurate simulation of collision events becomes increasingly important.

Main tools:

- Event generators: HERWIG, PYTHIA, etc.
- Numerical programs based on exact perturbative calculations:
  - Leading order (LO) calculations
  - Next-to-leading order (NLO) calculations
  - in the future: NNLO calculations

Event Generators

Event generators rely on a three-stage process:



Advantages:

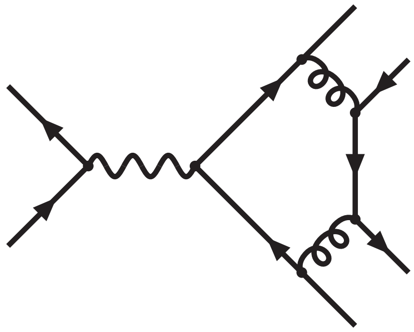
- Unweighted events
- Multiple collinear emission reasonably described

Disadvantages:

- Usually only matrix elements for $2 \to 2$ and $2 \to 3$ processes
- Usually only LO matrix elements
- Showering does not populate complete phase space

Numerical programs based on exact perturbative calculations

These numerical programs implement quantum corrections:



Advantages:

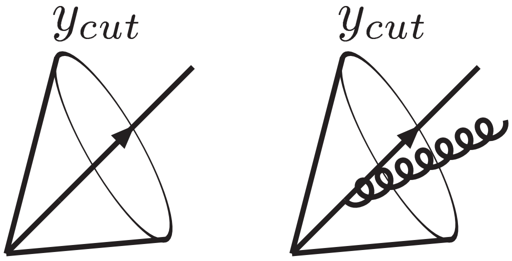
- Predictive power
- Accurate description if all jets are well-separated in phase space

Disadvantages:

- More elaborate and complex, only available up to a few partons
- Perturbation theory breaks down if observable depends on a large ratio of two scales

NLO programs

At next-to-leading order: Observables are calculated as a sum of two terms: One part living on a n-parton phase space, the other one living on a $(n+1)$-parton phase space.



Taken separately, the virtual corrections and the real emission part are divergent.

Phase-space slicing or subtraction method to cancel IR-divergences before any numerical integration is performed.

The NLO computer program calculates an observable then as a sum of two terms, each term being finite, but not necessarily positive definite.

Infrared-safe observables

Observables which do not depend on long-distance behaviour, are called infrared-safe observables and can reliably be calculated in perturbation theory.

In particular, it is required that they do not change value, if an infinitessimal soft particle is added or if one parton is replaced by two collinear ones.

Infrared-safe event shape observables, like thrust, jet broadening or aplanarity reveal more information about an event then the total cross section alone.

Example: Thrust

$$\mathbf{T} \quad = \quad \mathsf{max}_{\hat{n}} \frac{\sum\limits_i |\vec{p}_i \cdot \hat{n}|}{\sum\limits_i |\vec{p}_i|}$$

The most fine-grained look at hadronic events consistent with infrared safety is given by classifying the particles into jets.

Negative weights and double counting

Can one "generate events" according to NLO matrix elements ?

Double counting:
Gluon emission from hard scattering and parton showering.

Negative weights:
The n- and $(\mathrm{n}+1)$-phase space contributions are finite, but not necessarily positive definite.

Recent developments

- **Some thoughts and a possible road**
  C. Friberg, T. Sjöstrand

- **Subtraction method against double counting**
  J. Collins

- **Modified phase space slicing against negative weight for specific examples**
  B. Pötter, T. Schörner and M. Dobbs, M. Lefebre

- **A general algorithm for unweighted NLO parton events**
  S. W.

- **Resummation and parton showering**
  S. Mrenna

- **LO matrix elements and parton showers**
  S. Catani, F. Krauss, R. Kuhn, B. Webber

Unweighted events

The algorithm for a NLO partonic event generator:

Given a NLO-calculation, the algorithm yields a computer program, which generates a sequence of unweighted momentum configurations, each configuration containing either n or $n+1$ four-vectors, such that:

For any infrared-safe observable the average over these configurations coincides with the NLO calculation up to errors of a resolution variable y_{res}.

The algorithm is independent of the specific hard process; can be implemented on top of existing NLO programs.

As exclusive as possible,

as inclusive as necessary !

Basic idea

Consider a $(n+1)$-parton configuration:

- If all partons are separated by at least y_{res}:
  - Return this $(n+1)$ configuration.

- If some parton is unresolved at the scale y_{res}:
  - Integrate over the real emissions up to the scale y_{res}
  - Combine the result with the virtual corrections.

  - By choosing y_{res} large enough the sum will have a positive weight.

  - Return the n-parton configuration.

Technical challenges

Integrate efficiently the unresolved region !

- Modified version of the subtraction method. Non-vanishing subtraction terms only for $y < y_{res}$.

- Fourvectors of a $(n+1)$ parton configuration obtained by generating a n parton configuration first, then transforming this configuration into a $n+1$ configuration with a soft parton inserted.

  – Integration over the unresolved part involves only a three-dimensional integration.
  – $1/s_{ij}$-behaviour of the integrand absorbed into the measure.

- Metropolis algorithm to generate events according to a (complicated) probability distribution.

IR-divergences and subtraction method

The NLO cross section is given by

$$\sigma^{\mathrm{NLO}} = \int_{\mathbf{n}} d\sigma^{\mathrm{B}} + \int_{\mathbf{n}} d\sigma^{\mathrm{V}} + \int_{\mathbf{n+1}} d\sigma^{\mathrm{R}}$$

Within the subtraction method one add and subtracts a suitable piece:

$$\sigma^{\mathrm{NLO}} = \int_{\mathbf{n}} d\sigma^{\mathrm{B}} + \int_{\mathbf{n}} \left(d\sigma^{\mathrm{V}} + \int_{\mathbf{1}} d\sigma^{\mathrm{A}} \right) + \int_{\mathbf{n+1}} \left(d\sigma^{\mathrm{R}} - d\sigma^{\mathrm{A}} \right)$$

The approximation term is given by a sum over dipoles (S. Catani, M. Seymour):

$$\sum \mathcal{D}_{\mathbf{ij,k}}$$

The approximation term can still be integrated analytically if the region where it is subtracted, is restricted (Z. Nagy, Z. Trócsányi):

$$\sigma^{\mathrm{A}} = \sum \mathcal{D}_{ij,k} \Theta\left(y_{ij,k} < y_{res}\right)$$

$$y_{ij,k} = \frac{2p_i p_j}{2p_i p_j + 2p_i p_k + 2p_j p_k}$$

Generating soft insertions

Starting point: An algorithm to generate a "hard" n-parton configuration from m random numbers u_1, ..., u_m; e.g. sequential decay, RAMBO (Kleiss, Stirling and Ellis)

Generating soft insertions:

$$d\phi_{n+1} = d\phi_n \frac{ds_{as}ds_{sb}d\phi_s}{4(2\pi)^3 s_{ab}}$$

$$s_{as} = s_{ab}\left(\frac{s_{min}}{s_{ab}}\right)^{v_1}, s_{sb} = s_{ab}\left(\frac{s_{min}}{s_{ab}}\right)^{v_2}, \phi_s = 2\pi v_3$$

Reconstruct the four vectors of the $n+1$ parton configuration from these invariants.

$1/s_{ij}$-behaviour of the integrand absorbed into the measure:

$$d\phi_{n+1} = d\phi_n \frac{1}{4(2\pi)^2} \frac{s_{as}s_{sb}}{s_{ab}} \ln^2\left(\frac{s_{min}}{s_{ab}}\right) dv_1 dv_2 dv_3$$

Random numbers with a given distribution

Generating events according to a multi-dimensional probability density:

The LO matrix element for $e^+e^- \to qg\bar{q}$ is relatively simple

$$\frac{x_1}{x_2} + \frac{x_2}{x_1} + 2\frac{(1 - x_1 - x_2)}{x_1 x_2}$$

Can find a simple function which is everywhere larger and use the acceptance-rejection method.

Metropolis algorithm

In general we want to generate random vairables according to a probability density $P(x_1, ..., x_d)$, which not necesarrily factorizes.

The Metropolis algorithm: Start from a state $\vec{x} = (x_1, ..., x_d)$.

- **Generate randomly a new candidate $\vec{x}'$.**
- **Calculate $\Delta S = -\ln(P(\vec{x}')/P(\vec{x}))$.**
- **Accept the new candidate with probability $\min(1, e^{-\Delta S})$.**

The way new candidates are suggested is arbitrary, restricted only by ergodicity and detailed balance:

- **Each state must be reachable within a finite number of steps.**
- **Probability of suggesting x_2 given we are in state x_1 = probability of suggesting x_1 given we are in x_2.**

The algorithm

- Pick a set of random numbers $u_1, \dots, u_m, x, v_1, v_2, v_3$.
  - $u_1, \dots, u_m$ define the "hard" n-parton configuration.
  - x decides where the "soft" particle gets inserted.
  - v_1, v_2, v_3 + "hard" configuration + information where "soft" particle gets inserted yields the "soft" $(n+1)$ event.

- All $y_{ij,k} > y_{res}$?

  yes:

$$P' \sim \left| \mathcal{M}_{Born}^{n+1} \right|^2$$

  no:

$$\begin{aligned} P' \quad \sim \quad & \left| \mathcal{M}_{Born}^{n} \right|^2 \\ & + 2 \,\mathrm{Re}\, {\mathcal{M}_{Born}^{n}}^* \mathcal{M}_{1-loop}^{n} + \langle \mathcal{M}_{Born}^{n} \left| \mathbf{I} \right| \mathcal{M}_{Born}^{n} \rangle \\ & + \int dv_1 dv_2 dv_3 \left(\left| \mathcal{M}_{Born}^{n+1} \right|^2 - \sum \mathcal{D}_{ij,k} \right) \end{aligned}$$

- $\Delta S = -\ln(P'/P)$ and accept the new event with Metropolis propability $\min(1, e^{-\Delta S})$.

Outlook

Summary:

A general algorithm to generate unweighted events for NLO calculations in $\mathrm{e}^+\mathrm{e}^-$ annihilation.

Efficient integration over unresolved region on a point-by-point basis in hard phase space.

Metropolis algorithm to generate a sequence of events according to a (complicated) probability density.

Outlook:

Extension to initial-state partons or massive partons should be feasible.

Implementation for electron-positron annihilation.

Merging with parton showers.